



# COMMON PRE-BOARD EXAMINATION

## MATHEMATICS-Code No. 041

### Class-XII-(2025-26)



SET: 1

Time allowed: 3 Hrs.

Maximum Marks: 80

#### General Instructions:

Read the following instructions very carefully and follow them:

1. This Question Paper has 5 Sections A - E.
2. Section **A** has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 Case Based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks for two questions and 2 marks each for two sub questions for one question.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.

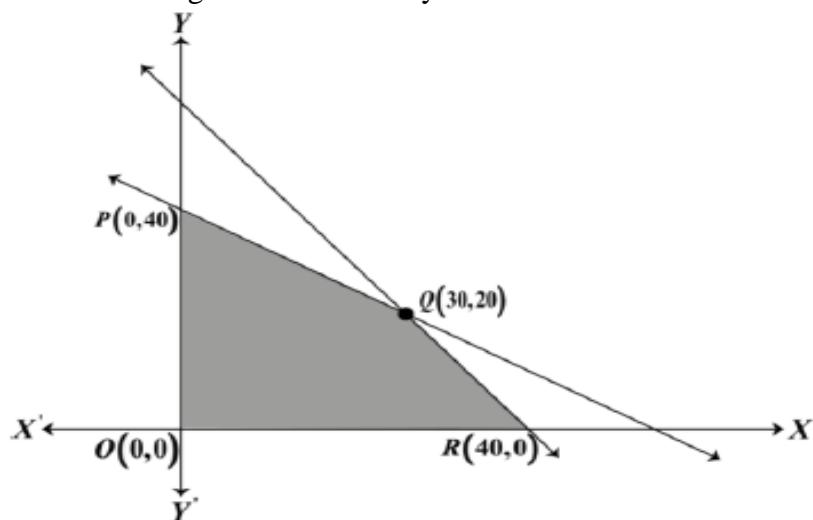
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#### SECTION-A [ 1 x 20 = 20]

(This section comprises of multiple-choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 20):

1. For the linear programming problem (LPP), the objective function is  $Z = 4x + 3y$  and the feasible region determined by a set of constraints is shown in the graph: [1]



Which of the following statements is true?

(A) Maximum value of  $Z$  is at  $R(40,0)$ .  
 (B) Maximum value of  $Z$  is at  $Q(30,20)$ .  
 (C) Value of  $Z$  at  $R(40,0)$  is less than the value at  $P(0,40)$ .  
 (D) The value of  $Z$  at  $Q(30,20)$  is less than the value at  $R(40,0)$ .

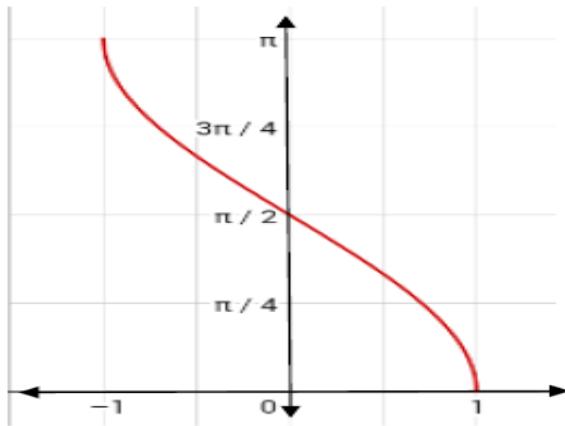
2. If  $E$  and  $F$  are two independent events such that  $P(E) = 2/3$ ,  $P(F) = 3/7$ , then  $P(E | \bar{F})$  is [1]  
 equal to :

(A)  $1/6$  (B)  $1/2$  (C)  $2/3$  (D)  $7/9$

3. If the rate of change of the volume of a sphere is twice the rate of change of its radius, [1]  
 then the surface area of the sphere is :

(A) 1 sq. unit (B) 2 sq. units (C) 3 sq. units (D) 4 sq. units

4. The graph drawn below depicts : [1]



(A)  $y = \sin^{-1}x$  (B)  $y = \cos^{-1}x$  (C)  $y = \operatorname{cosec}^{-1}x$  (D)  $y = \cot^{-1}x$

5. If  $m$  and  $n$ , respectively, are the order and the degree of the differential equation [1]

$\frac{d}{dx} \left[ \left( \frac{dy}{dx} \right)^4 \right] = 0$ , then  $m + n =$   
 (A) 1 (B) 2 (C) 3 (D) 4

6. If vector  $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$  and vector  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  then which of the following is [1]  
 correct?

(A)  $\vec{a} \parallel \vec{b}$  (B)  $\vec{a} \perp \vec{b}$  (C)  $|\vec{a}| < |\vec{b}|$  (D)  $|\vec{a}| = |\vec{b}|$

7. The lines  $\vec{r} = \vec{i} + \vec{j} - \vec{k} + \lambda(2\vec{i} + 3\vec{j} - 6\vec{k})$  and [1]

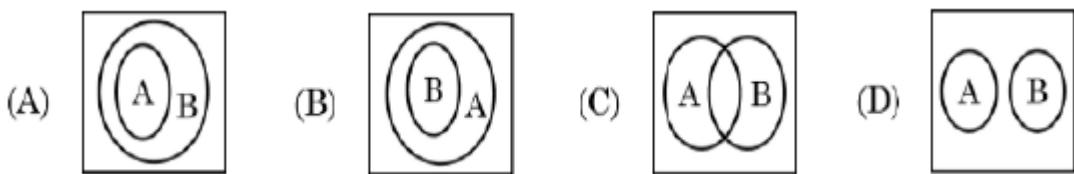
$\vec{r} = 2\vec{i} - \vec{j} - \vec{k} + \mu(6\vec{i} + 9\vec{j} - 18\vec{k})$  where  $\lambda$  and  $\mu$  are scalars) are

(A) Coincident (B) Skew (C) Intersecting (D) Parallel

8. If  $\int \frac{2^x}{x^2} dx = k \cdot 2^x + C$ , then  $k$  is equal to [1]

(A)  $\frac{-1}{\log 2}$  (B)  $-\log 2$  (C)  $-1$  (D)  $\frac{1}{2}$

9. If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following figure depicts the correct relation between A and B ? [1]



10. If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ ,  $|\vec{a}| = \sqrt{37}$ ,  $|\vec{b}| = 3$ , and  $|\vec{c}| = 4$ , then find the angle between  $\vec{b}$  and  $\vec{c}$ . [1]

(A)  $\pi/6$  (B)  $\pi/4$  (C)  $\pi/3$  (D)  $\pi/2$

11. If A and B are non-singular matrices of the same order with  $\det(A) = 5$ , then  $\det(B^{-1}AB)^2$  is [1]

(A) 5 (B)  $5^2$  (C)  $5^4$  (D)  $5^5$

12. The inverse of the matrix  $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$  is [1]

(A)  $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$  (B)  $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$  (C)  $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$  (D)  $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$

13. Value of the determinant  $\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix}$  is [1]

(A) 0 (B) 1/2 (C)  $\sqrt{3}/2$  (D) 1

14. The area of a triangle with vertices A, B, C is given by [1]

(A)  $|\vec{AB} \times \vec{AC}|$  (B)  $\frac{1}{2} |\vec{AB} \times \vec{AC}|$  (C)  $|\vec{AB} \times \vec{BC}|$  (D)  $\frac{1}{4} |\vec{AB} \times \vec{BC}|$

15. The value of  $\int_2^3 \frac{x}{x^2+1} dx$  is [1]

(A)  $\log 4$  (B)  $\log \frac{3}{2}$  (C)  $\frac{1}{2} \log 2$  (D)  $\log \frac{9}{4}$

16. If  $A = [a_{ij}]$  is a skew-symmetric matrix of order n, then [1]

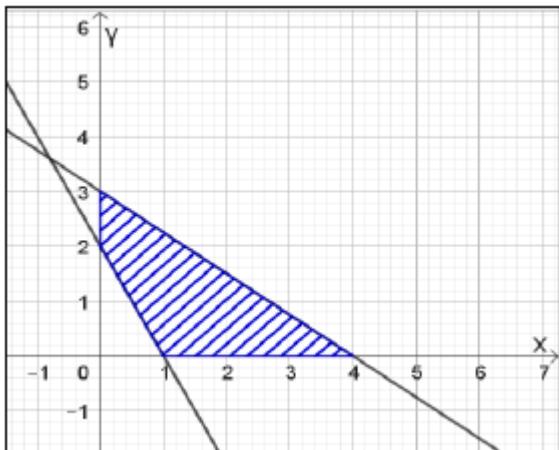
(A)  $a_{ij} = \frac{1}{a_{ji}}$  (B)  $a_{ij} \neq 0 \forall i, j$  (C)  $a_{ij} = 0$ , where  $i = j$  (D)  $a_{ij} \neq 0$ , where  $i = j$

17. If the area of the triangle with vertices  $(-3, 0)$ ,  $(3, 0)$  and  $(0, k)$  is 9 sq. units, then find the value/s of k. [1]

(A) 9 (B)  $\pm 3$  (C) -9 (D) 6

18. A student of class XII studying Mathematics comes across an incomplete question in a book. Maximise  $Z = 3x + 2y + 1$  Subject to the constraints  $x \geq 0$ ,  $y \geq 0$ ,  $3x + 4y \leq 12$ , [1]

He/ She notices the graph shown below for the said LPP problem, and finds that a constraint is missing in it:



Help him/her choose the required constraint from the graph.

The missing constraint is

(A)  $x+2y \leq 2$       (B)  $2x+y \geq 2$       (C)  $2x+y \leq 2$       (D)  $x+2y \geq 2$

**Questions numbered 19 and 20 are Assertion and Reason-based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:**

(A) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).  
 (B) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).  
 (C) Assertion (A) is true but Reason (R) is false.  
 (D) Assertion (A) is false but Reason (R) is true.

19.

**Assertion (A):**  $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$  is continuous at  $x = 5$  for  $k = 5/2$  [1]

**Reason (R):** For a function  $f$  to be continuous at  $x = a$ ,

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a).$$

20.

**Assertion (A):** The domain of the function  $\sec^{-1} 2x$  is  $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$ . [1]

**Reason (R):**  $\sec^{-1} 2x = -\frac{\pi}{4}$

### SECTION B [2 x 5 = 10]

**(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)**

21.

(a) Find the value of  $\sin^{-1} \left( \cos \left( \frac{33\pi}{5} \right) \right)$  [2]

**OR**

(b) Find the domain of  $\sin^{-1}(x^2 - 4)$

22. If  $\vec{\alpha}$  and  $\vec{\beta}$  are position vectors of two points P and Q respectively then find the position vector of a point R in QP such that  $QR = \frac{3}{2} QP$ . [2]

23. (a) Evaluate:  $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$  [2]

**OR**

(b) Evaluate:  $\int \frac{(x-3)}{(x-1)^3} e^x dx$

24. If  $e^y (x+1) = 1$ , prove that  $\frac{dy}{dx} = -e^y$ . [2]

25. Differentiate  $2^{\cos^2 x}$  w.r.t.  $\cos^2 x$ . [2]

### SECTION C [3 x 6 = 18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Consider the following Linear Programming Problem: [3]

Minimize  $Z = x + 2y$

Subject to  $2x + y \geq 3$ ;  $x + 2y \geq 6$ ;  $x, y \geq 0$ .

Show graphically that the minimum of Z occurs at more than two points.

27. Two dice are thrown. Two events A and B are defined as follows: [3]

$A = \{(x, y): x + y = 9\}$ ,  $B = \{(x, y): x \neq 3\}$ , where  $(x, y)$  denote a point in the sample space. Check if the events A and B are independent or mutually exclusive.

28. (a) An ant is moving along the vector  $\vec{l}_1 = \vec{i} - 2\vec{j} + 3\vec{k}$ . Few sugar crystals are kept along the vector  $\vec{l}_2 = 3\vec{i} - 2\vec{j} + \vec{k}$  which is inclined at an angle  $\theta$  with vector  $\vec{l}_1$ . Then find the angle  $\theta$ . Also find the scalar projection of  $\vec{l}_1$  on  $\vec{l}_2$ . [3]

**OR**

(b) Find the vector and the Cartesian equation of the line that passes through  $(-1, 2, 7)$  and is perpendicular to the lines  $\vec{r} = 2\vec{i} + \vec{j} - 3\vec{k} + \lambda(\vec{i} + 2\vec{j} + 5\vec{k})$  and  $\vec{r} = 3\vec{i} + 3\vec{j} - 7\vec{k} + \mu(3\vec{i} - 2\vec{j} + 5\vec{k})$ .

29. (a) Calculate the area of the region bounded by the curve  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the X- axis using Integration. [3]

**OR**

(b) In a rough sketch, mark the region bounded by  $y = 1 + |x + 1|$ ,  $x = -2$ ,  $x = 2$  and  $y = 0$ . Using integration, find the area of the marked region.

30. Find the intervals in which the function  $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$  is [3]

(a) strictly increasing (b) strictly decreasing

31. (a) If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ ,  $-1 < x < 1$ ,  $x \neq y$ , then prove that  $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ . [3]

**OR**

(b) If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ , find  $\frac{d^2y}{dx^2}$ .

**SECTION D [5 x 4 = 20 ]**

**(This section comprises of 4 long answer (LA) type questions of 5 marks each)**

32. The equation of the path traversed by the ball headed by the footballer is [5]  
 $y = ax^2 + bx + c$ ; (where  $0 \leq x \leq 14$  and  $a, b, c \in \mathbb{R}$  and  $a \neq 0$ ) with respect to an XY-coordinate system in the vertical plane. The ball passes through the points  $(2, 15)$ ,  $(4, 25)$ , and  $(14, 15)$ . Determine the values of  $a$ ,  $b$ , and  $c$  by solving the system of linear equations in  $a$ ,  $b$ , and  $c$ , using the matrix method. Also, find the equation of the path traversed by the ball.

33. Find the coordinates of the image of the point  $(1, 6, 3)$  with respect to the line [5]  
 $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ ; where  $\lambda$  is a scalar. Also, find the distance of the image from y-axis.

34. (a) Evaluate:  $\int_0^{3/2} |x \cdot \cos \pi x| dx$  [5]

**OR**

(b) Evaluate:  $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$

35. (a) Solve the differential equation:  $x^2 y dx - (x^3 + y^3) dy = 0$ . [5]

**OR**

(b) Solve the differential equation:  $\frac{dy}{dx} = \cos x - 2y$ .

**SECTION- E [4 x 3 =12 ]**

**(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)**

36. A city's traffic management department plans to optimise traffic flow by analysing the connectivity between various traffic signals. The city has five major spots labelled A, B, C, D, and E. The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C, and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.



The department wants to represent and analyze this data using relations and functions. [1]

Use the given data to answer the following questions: [1]

(i). Is the traffic flow reflexive? Justify. [2]

(ii). Is the traffic flow transitive? Justify. [2]

(iii) (a). Represent the relation describing the traffic flow as a set of ordered pairs. [2]

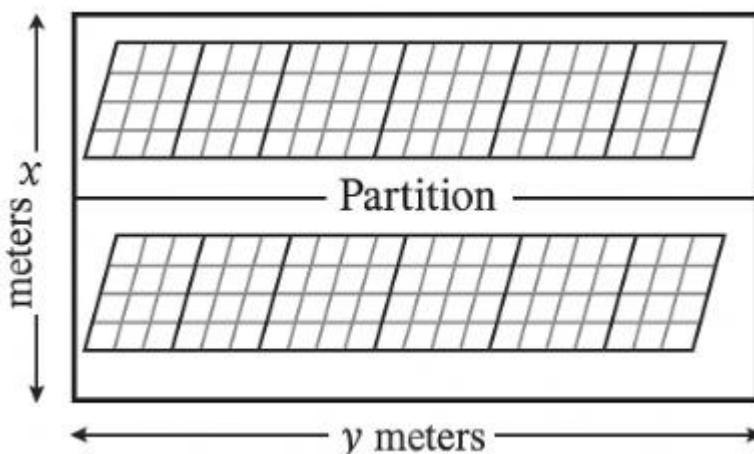
Also state the domain and range of the relation. [2]

**OR**

(b). Does the traffic flow represent a function? Justify your answer.

37. A technical company is designing a rectangular solar panel installation on a roof using 300 meters of boundary material. The design includes a partition running parallel to one of the sides, dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be  $x$  meters and the side parallel to the partition be  $y$  meters.



Based on this information, answer the following questions:

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of  $x$  and  $y$ . [1]

(ii) Write the area of the solar panel as a function of  $x$ . [1]

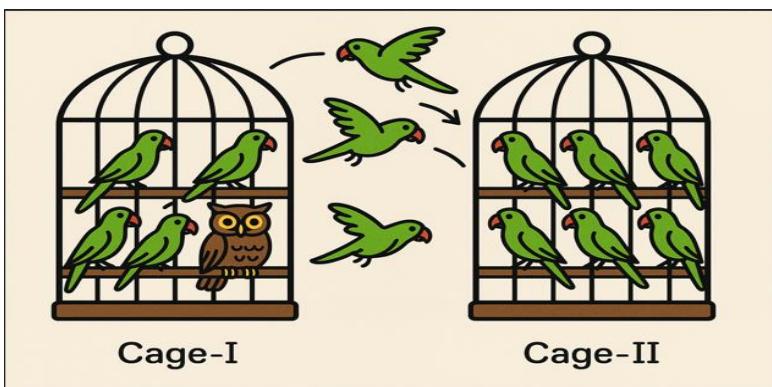
[2]

(iii)(a) Find the critical points of the area function. Use the second derivative test to determine critical points at the maximum area. Also, find the maximum area. [2]

**OR**

(b) Using the first derivative test, calculate the maximum area the company can enclose with the 300 meters of boundary material, considering the parallel partition.

38. Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage -II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I (simultaneously). Assume that all the birds have equal chances of flying.



Based on the above information, answer the following questions:-

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I. [2]

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II? [2]

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